Effect of Forecast Accuracy on Inventory Optimization Model

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Abstract

In this study, we examine the effect of forecast accuracy on the inventory costs incurred by a national retailer using a dynamic inventory optimization model. In the past, the retailer calculated weekly and monthly demand forecasts at a particular distribution center by dividing the annual demand with specific numbers which led to a consistently flat ordering model. This led the retailer to purchase items in bulk from their vendors which led to incurring unnecessary holding costs. The motivation for study is that this type of purchase behavior does not adequately prepare a supply chain for unexpected demand and thus might further deepen their inventory troubles. We introduced to this retailer an easy to deploy inventory model that uses the distribution of demand for each item along with the target service level among other constraints like the purchase capacity etc., to minimize the overall cost for each item. We then show the impact of inventory costs based on how accurate the demand forecast is.

Keywords: Dynamic Inventory Optimization, Forecast Accuracy, Wagner-Whitin Algorithm

Introduction

Companies use inventory as a buffer between supply and demand volatility. Handling optimal inventory level is important for retailers since too much inventory would imply too much capital struck in the supply chain, while too little inventory would prevent customer fulfillment to a satisfactory level. Further, mishandling inventory would not just affect the supply chain department but the overall KPIs of the firm.

Nowadays, large firms use ERP systems to aid them in their inventory decisions and these ERP systems integrate critical information about the supply chain, such as customer orders, warehouse capabilities, and demand forecasts to help managers make informed decisions (Fritsch,2017). Further, today firms are investing heavily into machine learning, Big Data Analytics, and the Internet of Things to bring more analytic power to operational decision support systems. For instance, a warehouse manager can now incorporate images as data inputs for intelligent stock management systems that can predict when the company should re-order (Marr, 2016).

By improving demand accuracy, companies can reduce the safety stock and free up more cash for other areas of the business. For business-to-consumer (B2C) supply chains, for example, a consumer goods company will forecast what stock keeping units a retailer will order. Also, demand management teams must collaborate with sales forces to accurately estimate the probability that a deal will close and what products the deal will include (Banker, 2013). Therefore, a company could reduce the inventory per square foot and turnover days to keep the cash flow.

Unfortunately, improving the demand forecasts is not enough. What really matters is how the company implements stocking and reorder points. In practice, other factors should be incorporated into the process design such as transportation costs. Recently, a Dutch adult beverage firm successfully reduced its freight cost by implementing dynamic order allocation which integrates customer order information and available stock levels. With more information, the company can make strategic decisions in optimizing cost while maximizing customer fulfillment (Banker, 2015).

Our research question in this study is, how does demand forecast accuracy translate to additional inventory costs when using a dynamic inventory optimization model for replenishment of spare-type items? We apply the retailer's own forecasts for a baseline. Then we build a dynamic optimization model using the Wagner-Whitin algorithm to increase the retailer's responsiveness to the market demand.

Our paper is organized as follows: We begun reviewing academic literature related to inventory management. Then, we discuss the type of data we have before diving into our methodology in getting better forecasts. We then used Wagner-Whitin algorithm to calculate the optimal ordering point(s). Lastly, we discuss the savings for the retailer as a function of the forecast accuracy before ending the discussion with further steps to improve this model.

Literature Review

We examined the academic literature and mainly investigated three things regarding formulating the optimized economic order quantity model.

1. Demand distribution: To make the model such as linear or non-linear model, understanding of demand's distribution is important.
2. Costs factors for the model: In the inventory system, classically three factors are considered as cost factors: i) replenishment cost, ii) holding cost, iii) shortage cost. Except for those classical factors, we investigated which other external factors could be considered when formulating our costs model.
3. Modeling with constraints: From our data sets, we have found several constraints, including minimum and maximum inventory level, and holding costs. We explored how to handle constraints from the literature.

Demand Distribution

In the supply chain field, an assumption of items is critical to build up the EOQ model. Most of the companies use a normality assumption for each item because it is the simplest way to build up the linear model. However, if intermittent demand occurs, the normal distribution is not plausible and exponential smoothing is used instead. In addition, as Bookbinder and Lordahl (1989) found, the bootstrap is superior to the normal approximation for estimating high percentiles of LTD distributions for independent data.

Below are several methods to estimate demand distribution. Classically, normal distribution, Poisson distribution, exponential smoothing method, Croston's method and bootstrap method are used in estimating the distribution of demand. Each method has pros and cons and has the most appropriate situation to be used. In addition, as Rehena Begum, Sudhir Kumar Sahu and Rakesh Ranjan Sahoo (2010) mention, each item's distribution can be considered. In supply chain field, snice items deteriorate, the Weibull distribution is widely used. Table 1 summarizes our findings based on the demand aspects.

Author	**Methods**	**Key Character**	**Advantages**
(Croston 1972)	Normal Distribution	Mean and Standard Deviation	The simplest way to make linear model with a normality assumption
(Ward 1978)	Poisson Distribution	Lambda	In a specific situation, it performs well
(Thomas R. Willemain*, Charles N. Smart, Henry F. Schwar 2004)	Exponential Smoothing	Robust forecasting method	Flexible over most of restrictions such as a normality assumption or Central Limit Theorem and performs well over Poisson Distribution
(Willemain et al., 1994; Johnston & Boylan, 1996)	Croston's Method	accurate forecasts of the mean demand per period	estimates the mean demand per period by applying exponential smoothing separately to the intervals between nonzero demands and their sizes
(Efron 1979)	Bootstrap Method	sampling with replacement from the individual observations	Ignore autocorrelation in the demand sequence and produce as forecast values only the same numbers that had already appeared in the demand history

Table 1: Literature on Demand Distribution

Cost factors

Classically, three factors are considered as cost factors: i) replenishment cost, ii) holding cost, and iii) shortage cost. Except for those classical factors, we investigated which other external factors could be considered in formulating our costs model. We mainly investigated in two ways: i) How those three cost factors can vary and ii) Whether other factors can be included in the modeling.

For example, Mark Ferguson, Vaidy Jayaraman and Gilvan C. Souza (2007) introduced a nonlinear way to handle the holding cost. A cumulative function of holding costs sometimes appears to be nonlinear, so that a different approach is more appropriate. Hoon Jung and Cerry M. Klein (2005) presented the optimal inventory policies for economic order quantity model with decreasing cost functions. It shows that classical costs can be interpreted in different ways as well as other costs factors being included in the model, such as interest rate or the deterioration of items. Table 2 below explains diverse ways to interpret cost factors for the model to minimize the cost.

Author	**Cost Factors**	**Key factors**	**Modeling explanation**
(Hoon Jung, Cerry M. Klein	Decreasing cost functions with	geometric	Constant demand and a fixed purchasing cost

2005)	economy of scale	programming (GP) techniques are used	
(Kun-Jen Chung, Leopoldo Eduardo Cárdenas-Barrón 2012)	Fixed backorder costs	Derivatives were used to find the optimal point	Two type backorders cost are considered: linear backorder cost (backorder cost is applied to average backorders) and fixed cost (backorder cost is applied to maximum backorder level allowed)
(J. Ray, KS. Chaudhuri 1997)	Stock-dependent demand, shortage, inflation and time discounting	Assumption of a constant purchasing cost becomes invalid in real situation	Explains relationships between cost factors and external factors. For example, the holding (or carrying cost) consists of opportunity costs and costs in the form of taxes, insurance and costs of storage.
(Mark Ferguson, Vaidy Jayaraman, Gilvan C. Souza 2000)	Nonlinear Holding Cost	cumulative holding cost is a nonlinear function of time	Appropriate with more significant for higher daily demand rate, lower holding cost, shorter lifetime, and a markdown policy with steeper discounts
(L.A. San-José, J. Sicilia, J. García-Laguna 2015)	Partial backordering and non-linear unit holding cost	backordering cost includes a fixed cost and a cost linearly dependent on the length of time for which backorder exists	Fixed cost which represents the cost of accommodating the item in the warehouse and a variable cost given by a potential function of the length of time over which the item is held in stock

Table 2. Diverse cost factors can be included in models

Constraints

In supply chain optimization, setting up constraints is critical. For example, our data set has multiple constraints, including minimum and maximum inventory level, minimum order quantity, and minimum service level to be achieved. Within those constraints, optimized economic order quantity should be calculated. However, within constraints, logic to find the optimal point are different for various models. For example, in linear modeling, the simplex method is widely used to find the optimal points. We examined which methods are being used to handle constraints and explanations and advantages of them, and specifically we focused on the service level that we need to achieve through for our business partner. Table 3 below explains how to find the optimal points with various constraints and what those models imply.

Author	**Constraints**	**Key factors**	**Modeling explanation**
(Sridar Bashyam, Michael C. Fu 1997)	Random Lead Time and a Service Level Constraint	Constraint simulation optimization	This paper considers the constrained optimization problem, where orders are allowed to cross in time
(Ilkyeong Moon, Sangjin Choi	Service Level	Stochastic inventory model	Service is measured here as the fraction of demand satisfied directly from stock

1994)			
(James H. Bookbinder, Jin Yan Tan 1988)	Lot-Sizing Problem with Service-Level Constraints	Time-varying demands	This paper describes deterministic version of problem, which is time-varying demands
(Wen-Yang Lo, Chih-Hung Tsai, Rong-Kwei Li 2000)	linear trend in demand	Demand rate of a product is a function of time	This study proposes a two-equation model to solve the classical no-shortage inventory replenishment policy for linear increasing and decreasing demand

Table 3. Literature on Constraints

Data

The data investigated in this study came from a regional retailer in the United States. The data set consisted of 87,053 observations of Part IDs, which includes diverse variables regarding the inventory system from a specific vendor at a specific distribution center. Table 4 provides a data dictionary of the features we had available for this study.

Variable	**Type**	**Description**
Date	Date	Date where inventory was examined at Remington DC
Lead Time _ Days	Numeric	When order was placed to when it was delivered to Remington distribution center
Product Group	Categorical	Parent directory of products. Product Group is composed of two groups 1. ELECTRICAL 2. NA
DC name	Categorical	Distribution Center Name
DC number	Categorical	Unique Distribution Center Number
Vendor number	Categorical	Unique Vendor Number
Part ID	Categorical	Unique parts identification
Part Description	Text	Detailed part description
Inventory on hand	Numeric	Week ending on Friday inventory on hand
Inventory on order	Numeric	Week ending on Friday inventory on order
Vendor request minimum order quantity	Numeric	A constraint
Current Purchase Price	Numeric	Current purchase price of each part
Units Shipped Year to Date	Numeric	Units shipped from a year ago to date

Units Shipped Quarter to Date	Numeric	Units shipped from a quarter ago to date
Units Shipped Week to Date	Numeric	Units shipped from a week ago to date
Demand Forecast Annual	Numeric	Forecasted annual demand
Demand Forecast Quarterly	Numeric	Forecasted quarterly demand
Demand Forecast Four Weekly	Numeric	Forecasted four weekly demand
Demand Forecast Weekly	Numeric	Forecasted weekly demand
Order Up To Level In Units	Numeric	A constraint
Minimum order level	Numeric	A constraint
Suggested Order Quantity Actual	Numeric	Suggested Order Quantity
Order Point Independent In Units	Numeric	Order Point Independent In Units
Item Class	Categorical	the groups that sell the most (As and Bs are faster moving)

Table 4: Data used in study

In addition to the features obtained in Table 4, we had actual outbound quantities data from the DC which was used for model validation.

Methodology

Exploratory Data Analysis

Figure 1 shows the actual outbound demand versus the retailer's currently demand forecast. This figure demonstrates the major cause of their high inventory costs they were facing.

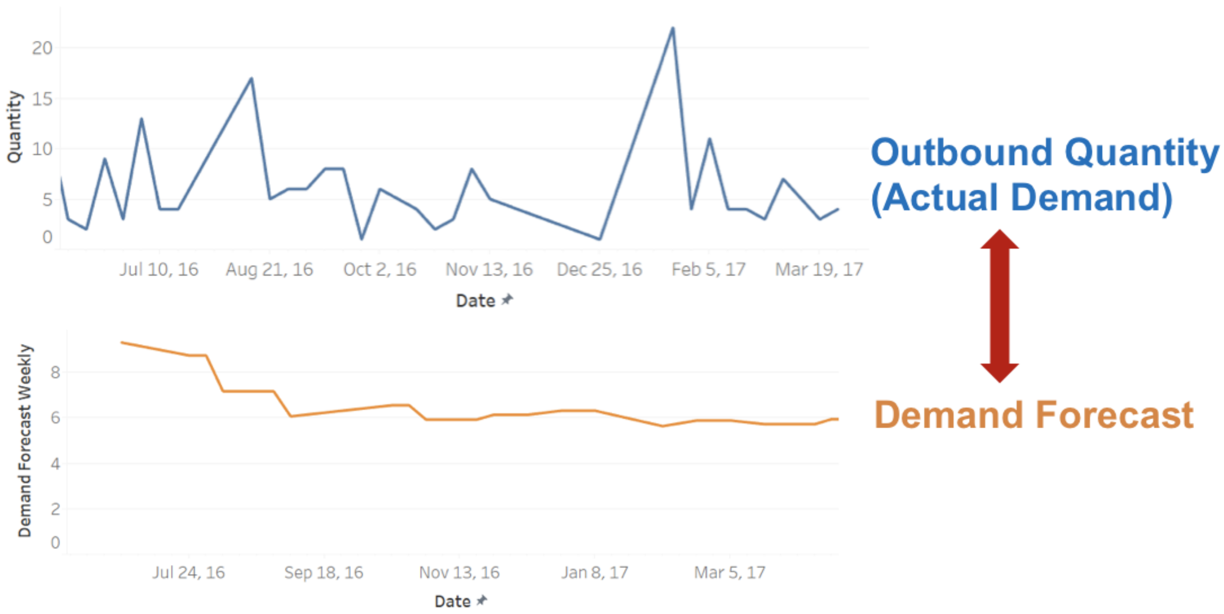

Figure 1: Actual Demand vs Demand Forecast

We began observing large differences between the retailer's forecast and the actual demand, which translated into high inventory management costs in form of underage costs and holding costs. Further we see different patterns in demand that led us to cluster the items into different clusters to deal with them differently. Our pipeline for the process is shown in Figure 2.

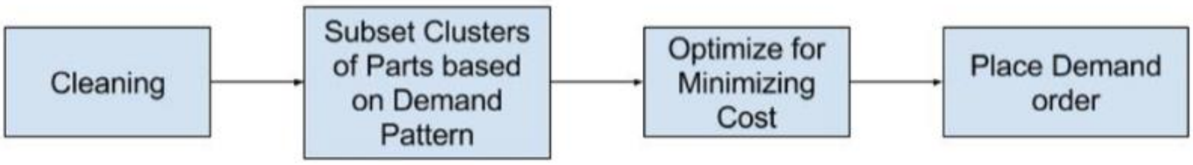

Figure 2: Data Processing Pipeline.

Once the data was cleaned and the subset clusters were identified, we used the retailer's forecast as a baseline and built three forecast models of varying accuracy to feed into the optimization model.

Models

Periodic Review Model

Our baseline optimization model was a standard Economic Order Point (EOP) model since the retailer had several constraints on the number of units they can order reducing their flexibility. Thus, an optimization on when they can order was reasonable. Figure 3 below explains the flow chart for an EOP model.

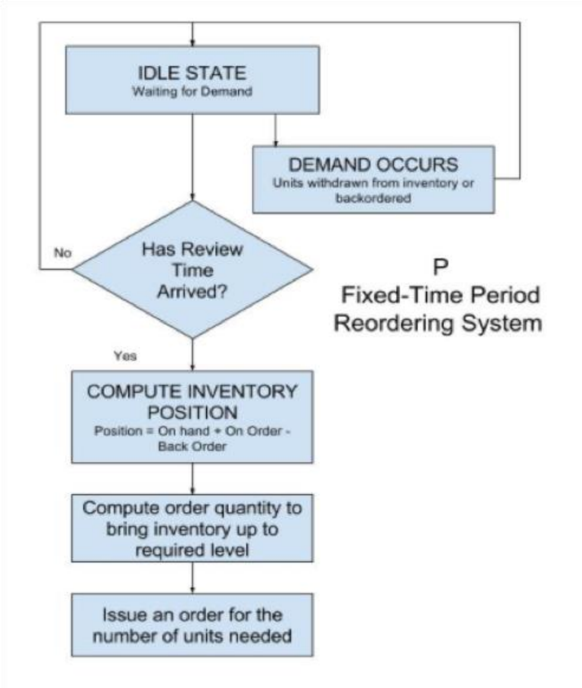

Figure 3: Economic Order Point

Following the above flowchart, EOP is a continuous review process where we check if we reached a pre-determined re-order point after every order fulfillment and we trigger an order if we did reach a re-order point.

Exponential Smoothing with Trend

Exponential Smoothing refers to an averaging method that weighs the most recent data more strongly. This is useful if the data changes as a result of seasonality (or pattern) instead of a random walk. Mathematically, this can be written as

$$F_{t+1} = \propto D_t + (1 - \propto) F_t$$

where F_{t+1} = Forecast for the next time period

D_t = Actual Demand at time t

F_t = Forecast at time t

$\propto$ = Weighing factor referred to as a smoothing parameter

This can be enhanced by adding a trend adjustment factor to incorporate the trend into the equation.

$$AF_{t+1} = F_{t+1} + T_{t+1} and$$

$$T_{t+1} = \beta(F_{t+1} - F_t) + (1 - \beta)T_t$$

where A_t = Adjusted Forecast for time t+1

T_t = Last period's trend factor and

β = Smoothing parameter for trend

For this model, people tend to use Mean Average Deviation (MAD) as the metric of determination.

Dynamic Programming

We used the Wagner-Whitin algorithm to help decide the time of the re-order point. The Wagner-Whitin Algorithm divides the n-period optimization problem to a series of sub-problems and each sub-problem is solved and used in solving the next sub problem. For example, if the decision maker wishes to plan his reorder point for six periods, the model divides his 6-period problem into six single period problems. On week 1, the decision maker has six options, order for all six weeks, order for the first five weeks, etc. all the way down to just ordering for just the first week. Choosing the option that minimizes the sum of holding cost and ordering cost would be the best decision for the decision maker. The series of such recurrent decisions made for all six periods gives us the overall minimum cost path. Mathematically,

Step 1: t = 1, $z_t^* = 0$

Step 2: t = t+1. If t > T+1, stop. Otherwise go to step 3.

Step 3: For all t' = 1, 2, ..., t - 1,

$$c_{t',t} = A_{t'} + c_{t'}(D_{t'} + \ldots + D_{t-1}) + h_{t'}(D_{t'+1} + \ldots + D_{t-1}) + h_{t'+1}(D_{t'+2} + \ldots + D_{t-1}) + \ldots + h_{t-2}D_{t-1}$$

Step 4: Compute

$$z_t^* = \min_{t'=1,2,\ldots,t-1} \{z_{t'}^* + c_{t',t}\}$$

Step 5: Compute

$$p_t^* = \arg\min_{t'=1,2,\ldots,t-1} \{z_{t'}^* + c_{t',t}\}$$

that is, choose the period t' that minimizes $z_{t'}^* + c_{t',t}$

Step 6: Go to step 2.

The optimal cost is given by z_{T+1}^*

The optimal set of periods in which ordering/production takes place can be obtained by backtracking from p_{T+1}^*

Results

Based on the above model, we calculated the total cost of inventory management for different products and it is observed that we are able to save about 13.7 % using the actual demand +/- a random value of 1 unit (which gives us an accuracy rate of 85%). We can save up to 20% in inventory costs using the dynamic model developed.

We also added an additional parameter into the equation that penalized the retailer for understocking. For example, there was only opportunity cost previously, but we considered the possibility of backordering which would inflict 1.5 times the cost for the retailer, and this caused an additional saving of 20% because of the optimization.

Conclusion

As discussed, forecasting and optimization go hand in hand when it comes to inventory optimization. Accurate forecasts lead to lower costs. Figure 4 shows the relationship we found when comparing the demand forecasting model's accuracy and the decisions the inventory decisions that would occur because of the model.

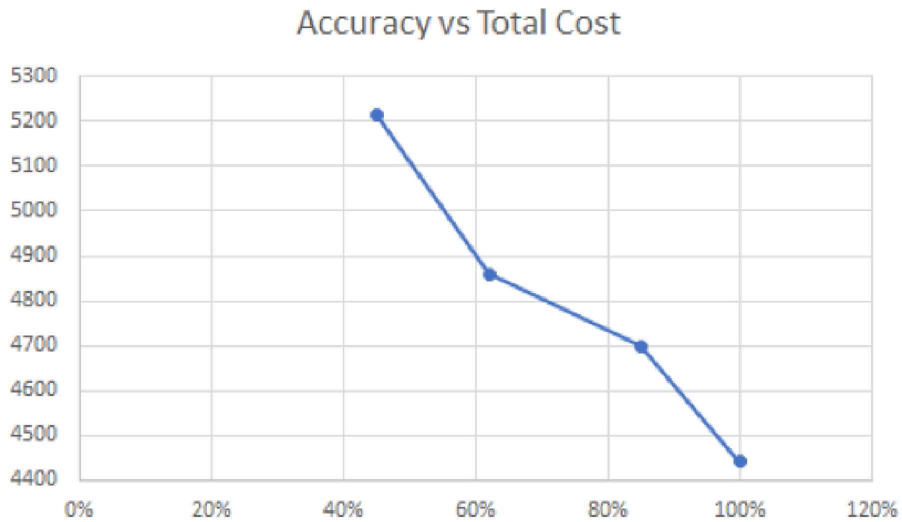

Figure 4: Model accuracy versus total costs ($)

In summary, we compared the results from our model for the three predicted demands to see which one performed better based on the accuracy of demand. We used total cost incurred during the period under consideration including the cost of stock-outs as a metric to compare the performance. We found that the client's predicted demand had the highest total cost associated with it due to poor accuracy. Our attempt to use the Exponential Smoothing technique to simulate predicted demand did not yield an accurate forecast. As the model with highly accurate predicted demand showed least total cost, we suggest the client to improve their accuracy of demand forecast, and thus realize better inventory performance.

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